AMPLIFICATION AND TEMPERATURE INSTABILITY EFFECTS IN A THERMOCOUPLE WITH STRONGLY TEMPERATURE-DEPENDENT THERMOELECTRIC PARAMETERS

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It is shown that the temperature dependence of the branches of a thermocouple can produce unstable operating regimes and steep performance curves.

Peltier thermocouples are used not only for thermoelectric cooling and as current sources, but also in signal-conversion applications [1, 2]. We now show that when semiconductors with highly temperature-dependent thermoelectric parameters (electrical conductivity and thermal emf) are used as materials for thermocouples, the slope of the performance curves of the device can increase considerably. Moreover, under certain conditions a thermocouple can acquire a temperature instability, which can also be utilized in various devices such as storage elements.

We consider the stability of the state of a Peltier thermocouple, to which a fixed voltage U is applied. The working junction is characterized in the equilibrium state by the temperature T, which is either higher or lower than the temperature T_0 of the external junctions, depending on the sign and magnitude of U. The relationship between the voltage U, the current I through the thermocouple, and the temperature difference $\Delta T = T - T_0$ is described by the equation

$$U = IR - \alpha\Delta T. \tag{1}$$

The quantity of heat released at the working junction represents the sum of the Peltier heat, the heat transferred as a result of the temperature difference between the contacts, and half the Joule heat:

$$Q = \frac{1}{2} I^2 R - \Pi I - K\Delta T, \tag{2}$$

where $\Pi = \alpha T$. The direction of the current is said to be positive when it cools the working junction by virtue of the Peltier effect. For simplicity we neglect the Thomson effect and heat transfer between the working junction and the environment.

The net heat Q = 0 in the steady state. Let us suppose that the temperature of the working junction increases by a small increment dT as a result of fluctuations or a weak thermal signal. This changes the thermoelectric parameters of the semiconductor, which we assume are functions of the mean temperature $\bar{T} = (T + T_0)/2$, and it changes the current I, causing a heat difference dQ to occur at the working junction. If the heat release dQ is also positive for dT > 0, the temperature of the working junction will increase further, causing the quantity of heat to increase, and so on. A temperature instability thus sets in if

$$\frac{dQ}{dT} > 0. \tag{3}$$

This derivative is evaluated according to Eqs. (1) and (2) under the condition dU = 0. We introduce dimensionless quantities for convenience:

$$\mathcal{I} = I \frac{R}{\alpha T}, \tag{4}$$

$$r_A = \frac{T}{A_{T=\bar{T}}} \left(\frac{dA}{dT} \right)_{T=\bar{T}}. \tag{5}$$

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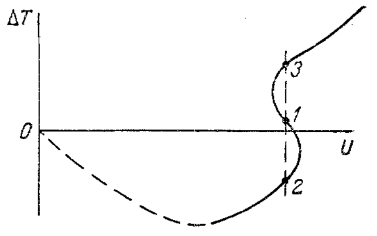

Fig. 1. Qualitative dependence of ΔT on U in the case of a temperature instability for $\mathcal{I} \approx 2$: 1) unstable state; 2, 3) stable states.

The parameters r_α and r_R (A denotes either of the quantities α, R) characterize the temperature dependence of α and R and are approximately equal to the logarithmic derivatives

$$r_A \simeq \frac{d \ln A}{d \ln T}. \tag{5a}$$

Calculations yield

$$\frac{dQ}{dT} = -\frac{\alpha^2 T}{R} \Delta, \tag{6}$$

$$\Delta = \left(1 + \frac{1}{ZT}\right) + r_\alpha \left[\frac{\mathcal{I}}{2}\left(1 - \frac{\Delta T}{T}\right) + \frac{\Delta T}{2T}\right] - r_R \frac{\mathcal{I}}{2}\left(1 - \frac{\mathcal{I}}{2}\right). \tag{7}$$

According to Eq. (3), the instability criterion has the form $\Delta < 0$. Since the first term in Eq. (7) is positive, instability can set in only for sufficiently large absolute values of r_α and r_R.

The most important case for practical applications is the case of small temperature differences ΔT. This situation arises for small currents of either sign ($|\mathcal{I}| \ll 1$), and also for $\mathcal{I} \approx 2$ when the Peltier effect at the working junction is compensated by the input of Joule heat. In the latter case, instability is achieved at $T \approx T_0$ if the thermal emf drops rapidly with the temperature:

$$r_\alpha < -\left(1 + \frac{1}{ZT}\right). \tag{8}$$

If the state with $\mathcal{I} \approx 2$ is unstable, the dependence of ΔT on the voltage U represents an S-shaped curve (see Fig. 1), where one value of U corresponds to an unstable state 1 at $T \approx T_0$ and two stable states 2 and 3 at $T > T_0$ and $T < T_0$, respectively. Other performance curves of the system, the volt–ampere characteristic (VAC) in particular, are equally complicated. The calculation of the differential resistance of the thermocouple for a slowly varying voltage yields the equation

$$R_{\text{dif}} = \frac{dU}{dI} = R \frac{\Delta}{\left(\mathcal{I} + \frac{1}{ZT}\right) + r_\alpha \frac{\mathcal{I}}{2} - r_R \left(\frac{\mathcal{I}}{2}\right)^2}. \tag{9}$$

The resistance R_{dif} is negative for $\Delta < 0$. A negative differential resistance is also observed when the denominator of Eq. (9) is less than zero. This case corresponds to a temperature instability of the thermocouple at a fixed current I.

A system with two different stable states can be utilized as a storage element. A symmetrical system of two identical parallel-connected thermocouples carrying a fixed total current 2I is interesting from this point of view. An analysis of this system shows that the state with equal currents $I_1 = I_2 = I$ through the thermocouples is unstable, and two mutually symmetrical states $I_1 < I_2$ and $I_1 > I_2$ are stable when the criterion $\Delta < 0$ holds [or in the special case $\mathcal{I} \approx 2$ of the criterion (8)]. This device is a thermoelectric analog of the trigger circuit.

If the instability criterion is not satisfied, but the system is close to instability (Δ is small and positive), various performance curves of the system can be very steep, and this feature can prove useful in practice. Consider, for example, the response of the system to a small heat flux W admitted to the working junction. For a fixed voltage, the derivative

$$\frac{dI}{dW} = \frac{R}{\alpha^2 T \Delta} \tag{10}$$

can be very large as instability is approached, i.e., small heat fluxes induce a large variation of the temperature of the working junction, which can be measured by means of a thermistor, as proposed by Stil'bans, Sher, et al. [2].

The current variation in the thermocouple is also large when the working junction is exposed to a heat flux W. If the voltage U is distributed between the thermocouple and a load resistance R_L connected in series with it, the voltage drop across R_L is described by the derivative

$$\frac{dU_L}{dW} = \frac{R_L}{\alpha T} \frac{1 + r_\alpha \frac{\Delta T}{2T} - r_R \frac{\mathcal{I}}{2}}{\Delta}, \tag{11}$$

where the quantity Δ is equal to the following in the presence of the load:

$$\Delta = \left(1 + \mathcal{I}\frac{R_L}{R} + \frac{1}{ZT}\,\frac{R + R_L}{R}\right) + r_\alpha\left[\frac{\mathcal{I}}{2}\left(\frac{R + R_L}{R} - \frac{\Delta T}{T}\right) + \frac{\Delta T}{2T}\right] - r_R \frac{\mathcal{I}}{2}\left(1 - \frac{\mathcal{I}}{2}\,\frac{R - R_L}{R}\right). \tag{12}$$

If the instability criterion $\Delta < 0$ holds for $R_L = 0$, the addition of a load permits the quantity Δ to be made positive and small, so that the system becomes sensitive to small heat fluxes.

It can be shown analogously that the system becomes sensitive to small variations of the voltage U and the temperature T_0 of the external contacts as instability is approached. Thus, the sensitivity of various signal converters can be increased substantially with the application of semiconductor thermoelectric materials with highly temperature-dependent parameters.

The practical implementation of these devices requires the development of a new group of thermoelectric materials capable of satisfying the criterion of instability or smallness of Δ. Two groups of materials can be cited, where the thermal emf decreases rapidly with increasing temperature with a sufficiently high value of Z preserved in a certain temperature range.

The first group comprises Bi_2Te_3 solid solutions in the temperature range where minority carriers begin to appear. Using the experimental results of Kutasov et al. [3], we can verify the fact that in p-Bi_2Te_3 with a hole density $\sim 10^{19}$ cm^{-3} in the temperature range around 400°K the quantity Δ for $\mathcal{I}=2$ is small for one of the samples. If it were also possible to find an n-type material with analogous properties, the parameters of the thermocouple would be $r_\alpha \simeq -4$ and $ZT \simeq 0.31$, i.e., for I = 2, according to Eq. (7), $\Delta \simeq 0.2$, whereas in the absence of a strong temperature dependence of α (or for small currents I) in superior materials, $ZT \approx 1$ and $\Delta \approx 2$. Thus, the sensitivity of heat-flux sensors, which is inversely proportional to Δ according to Eq. (10), can be increased by an order of magnitude through the temperature dependence of the thermal emf.

The second group of materials in which the instability criterion can be achieved comprises thermoelectric materials with a phase transition. For example, abrupt variations of R and α [4, 5], including intervals of rapid drops in α [4], are observed near the phase transition in Cu_2Se, which has a high thermoelectric efficiency.

NOTATION

U, voltage on thermocouple; I, current; R, resistance of thermocouple; α, thermal emf; T, temperature of working junction; T_0, temperature of external contacts; $\bar{T}$, mean temperature; $\Delta T = T - T_0$, temperature difference; K, thermal conductivity of thermocouple; Π, Peltier coefficient; Z, thermoelectric efficiency; Q, quantity of heat released at working junction; $\mathcal{I} = IR/\alpha T$, dimensionless value of current; r_α, r_R, parameters characterizing the temperature dependence of α and R, defined by Eq. (5); Δ, instability criterion, defined by Eq. (7) or (12); R_{dif}, differential resistance of thermocouple; W, external heat flux admitted to working junction; R_L, load resistance; U_L, voltage across load.

LITERATURE CITED

1. A. A. Bugaev, B. P. Zakharchenya, L. G. Pyzhov, et al., Fiz. Tekh. Poluprovodn., 13, No. 7, 1446-1447 (1979).
2. Yu. I. Ageev, A. É. Bilyalov, L. S. Stil'bans, and É. M. Sher, Pis'ma Zh. Tekh. Fiz., 7, No. 17, 1058-1061 (1981).

3. V. A. Kutasov, B. Ya. Moizhes, and I. A. Smirnov, Fiz. Tverd. Tela (Leningrad), 7, No. 4, 1065-1077 (1965).
4. Z. Ogorelec and B. Celustka, J. Phys. Chem. Solids, 27, No. 3, 615-617 (1966).
5. T. Ishikawa and S. Miyatani, J. Phys. Soc. Jpn., 42, No. 1, 159-167 (1977).

SYNTHESIS OF THERMOSTATING DEVICES. III. MINIMIZATION OF THE ERROR OF THERMOSTATING

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The article suggests a method of choosing the design parameters for a heater-type thermostat. Approximate formulas are obtained for determining the dynamic error in on-off control, and the problem of its minimization is dealt with.

Statement of the Problem. Synthesis of the optimal design of a thermostating device presupposes the choice of design and regime parameters ensuring the minimum of some target function which also includes quality indices of the device to be designed. Among these indices may be the static and dynamic errors of thermostating, the time required to attain the operating regime, characteristics of weight and overall dimensions, etc. The problem of choosing the optimal design and regime parameters can be solved on the basis of various mathematical models of thermostats dealt with in [1]. If the mathematical model is not suitable for establishing a fairly simple analytical dependence of the target function on the variable parameters, then for the sake of optimal choice of parameters numerical methods of optimization have to be used or the problem has to be solved by the method of nonformalized scanning of the variants. Often it is found that both approaches are ineffective unless the nature of the dependence of the quality indices on the variable parameters had been previously investigated and preliminary evaluations of the ranges had been obtained within which the optimal values of the parameters lie. It is advisable to carry out such a preliminary analysis with the aid of the simplest mathematical models that yield analytical expressions for the investigated quality indices.

The present article examines the problem of minimizing the static and dynamic errors of thermostating by means of optimal choice of parameters of a heater-type thermostat on the basis of models with lumped parameters. As elements the simplest model contains the object of thermostating 1, the chamber 2 with controlled power, and sensor 3 mounted on the chamber. We denote the temperature of these elements by t_1, t_2, and t_3, and we write the system of equations of thermal balance [1]:

$$P_1 = C_1 \frac{dt_1}{d\tau} + \sigma_{12}(t_1 - t_2) + \sigma_{1c}(t_1 - t_c), \tag{1}$$

$$P_2 = C_2 \frac{dt_2}{d\tau} + \sigma_{12}(t_2 - t_1) + \sigma_{23}(t_2 - t_3) + \sigma_{2c}(t_2 - t_c), \tag{2}$$

$$0 = C_3 \frac{dt_3}{d\tau} + \sigma_{23}(t_3 - t_2) + \sigma_{3c}(t_3 - t_c). \tag{3}$$

The power P_2 of the final control element situated on the chamber depends on the temperature of the sensor t_3. Confining ourselves to considering on-off control, we write this dependence in the form

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